Math 119 B, Midterm

ODEs and Dynamical Systems

Final is due in class Thursday March 14th.

1. Do the qualitative analysis of the equation,
$$\ddot{x} - x + x^2 = 0,$$
   (a) Find the stationary solutions.
   (b) Determine the stability of the stationary solutions.
   (c) Draw the phase portrait of the equation.
   (d) Use the phase portrait and (a) and (b) to identify four different types of solutions of the equation and describe their qualitative behaviour.
   (e) How would you go about computing the homoclinic orbit?

2. Find the (linear) stable and unstable manifolds of the stationary solution of the system
$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$
at the origin.

3. Show that the equation
$$\ddot{x} + \delta\dot{x} + x + x^3 = 0,$$
has global solutions that exist for all time, $\delta > 0$.

4. Show that the origin is a stable solution of the equation
$$\ddot{x} + \delta\dot{x} + x - x^2 = 0.$$
Then find the energy of the equation
$$\ddot{x} + x - x^2 = 0,$$
and use it to show that for this second equation the origin is (Lyapunov) stable.

5. Use the Poincaré map to describe the solutions of

$$\ddot{x} + \delta \dot{x} - x + x^2 = \epsilon \cos(\omega t).$$

In particular

(a) Draw the phase portrait.

(b) Describe the trajectories close to the origin.

(c) Describe what happens to the homoclinic orbit.