Math 119 A, Midterm

ODEs and Dynamical Systems

1. Write the ODE as a first order system

$$\frac{d^4x}{d^4t} + 5\frac{d^3x}{d^3t} + 7\frac{d^2x}{d^2t} + 2\frac{dx}{dt} + 4x = 0.$$

2. Solve the 2x2 ODE systems $\dot{x} = Ax$ by finding the eigenvalues and eigenvectors and exponentiating the matrix $P^{-1}AP$,

   (a)
$$A = \begin{pmatrix} 2 & 6 \\ 1 & -3 \end{pmatrix},$$

   (b)
$$A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix},$$

   (c)
$$A = \begin{pmatrix} 1 & 5 \\ -\frac{1}{2} & 2 \end{pmatrix}.$$

3. Draw the phase portraits for the three ODEs above and classify the flow as sinks, sources, centers, etc.

   (a)
$$A = \begin{pmatrix} 2 & 6 \\ 1 & -3 \end{pmatrix},$$

   (b)
$$A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix},$$

   (c)
$$A = \begin{pmatrix} 1 & 5 \\ -\frac{1}{2} & 2 \end{pmatrix}.$$

4. Suppose that a matrix A has one complex eigenvalue $\lambda = a + ib$, with $a < 0$ and one real eigenvalue $\lambda = c < 0$ with (algebraic) multiplicity two but only one eigenvector. What does the Jordan normal form $P^{-1}AP$ of the matrix look like and what is the solution $y(t)$ of the ODE

$$\dot{x} = Ax$$

in the coordinates $y = P^{-1}x$? What does the phase portrait look like?

5. Solve the 3x3 ODE systems $\dot{x} = Ax$ by finding the eigenvalues and eigenvectors and exponentiating the matrix $P^{-1}AP$,

(a)

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{pmatrix},$$

(b)

$$A = \begin{pmatrix} 2 & 1 & 0 \\ -9 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix},$$

(c)

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{pmatrix}.$$

6. Find the subspaces E^s, E^c, E^u of the ODE

$$\dot{x} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} x,$$

and find vectors x^s, x^c, x^u in each subspace. What happens to the solutions with these vectors as inital data as $t \to \infty$?

7. Find the Jordan normal form of the matrix and solve the IVP,

$$A = \begin{pmatrix} 0 & -1 & -2 & -1 \\ 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$