1. Integrate the functions
   
   (a) \[ \int \left( \frac{1}{x^2} - \frac{1}{x^3} \right) dx \]

   (b) \[ \int \left( 2e^u + \frac{6}{u} + \ln(2) \right) du \]

   (c) \[ \int x \cos(1 - 3x^2) dx \]

2. Solve the differential equation
   
   (a) \[ \frac{dy}{dx} = \frac{x + 1}{\sqrt{x}}, \quad y(5) = 4 \]

   (b) \[ \frac{dy}{dx} = e^{-x} + x^2, \quad y(0) = 4 \]

   \[ \frac{dy}{dx} = \sin(x) \cos(x), \quad y(0) = -\frac{1}{4} \]

3. How quickly will $2,000 grow to $5,000 when invested at an annual interest rate of 8% if compounded
   
   (a) Quarterly
   
   (b) Continuously
Hint: Recall that if the principal $P$ is invested at an annual rate of $r\%$ com-
pounded every $q$ months, then the total amount (principal and interest) ac-
cumulated after $s$ years is given by the formula

$$A(r) = P \left( 1 + \frac{0.01r}{q} \right)^{qs}$$

4. The number of bacteria in a certain culture grows exponentially. If 5,000
bacteria were initially present and 8,000 were present 10 minutes later, how
long does it take for the number of bacteria to double?

5. Compute the indefinite integrals

(a) $$\int t(t^2 + 1)^5 dt$$

(b) $$\int x^5 e^{1-x^6} dx$$

6. Compute the indefinite integrals

(a) $$\int \sec^2(x) dx$$

(b) $$\int \cos^2(2x) dx$$

7. Compute the definite integrals

(a) $$\int_{-1}^{1} e^{-x}(4 - e^x) dx$$

(b) $$\int_{1}^{2} \frac{x^2}{(x^3 + 1)^2} dx$$
8. Compute the left Riemann integral of \( f(x) = x^2 \), over the interval \( x \in [0, 1] \), by using 5 subintervals. Draw your approximation to the area.

9. Find the area of the region between the graph of the functions \( f(x) \) and the \( x \) axis, over the interval \([a, b]\). Draw the regions.
   (a) \( f(x) = (3x + 4)^{1/2} \), over \([0, 4]\)
   (b) \( f(x) = xe^{-x^2} \), over \([0, 3]\)
   (c) \( f(x) = \frac{1}{x} \), over \([1, e^2]\)

10. Find the area between the curves in the given interval, draw the curves and sketch the area,
   (a) \( \int_0^{\ln(2)} e^x \, dx \)
   (b) \( f(x) = e^x \), \( g(x) = e^{-x} \), \( 0 \leq x \leq \ln(2) \)
   (c) \( f(x) = \frac{1}{x^2} \), \( g(x) = x \), \( h(x) = \frac{8}{x} \)

11. At a certain factory, the marginal cost is \(6(q - 5)^2\) dollars per unit when the level of production is \( q \) units. By how much will the total manufacturing cost increase if the level of production is raised from 10 to 13 units?

12. A protein with mass \( m \) (grams) disintegrates into amino acids at a rate given by
    \[
    \frac{dm}{dt} = \frac{-15t}{t^2 + 5}
    \]
    What is the net change in mass of the protein during the first 4 hours?
13. It is estimated that \( t \) years from the beginning of the year 2012, the demand for oil in a certain country will be changing at the rate \( D'(t) = (t + 2r)^{-1} \) billion barrels per year. Will more oil be consumed (demanded) during 2013 or during 2016? How much more?

14. Compute the indefinite integrals
   
   (a) \[ \int x(\ln(x))^2 \, dx \]

   (b) \[ \int x \sin(2x) \, dx \]

   (c) \[ \int \exp(2x) \cos(3x) \, dx \]

15. A tree has been transplanted and after \( x \) years is growing at the rate of

   \[ h'(x) = 0.5 + \frac{1}{(x + 1)^2} \] meters per year. By how much does the tree grow during the second year.

16. Money is transferred continuously into an account at the rate of \( 5,000e^{0.015t} \) dollars per year at time \( t \) (years). The account earns interests at the annual rate of 5% compounded continuously. How much will be in the account at the end of 3 years?

17. The rate at which an epidemic spread through a community with 2,000 susceptible residents is jointly proportional to the number of residents who have been infected and the number of susceptible residents who have not. Express the number of residents who have been infected as a function of time (in weeks), if 500 residents had the disease initially and 855 resident had been infected by the end of the first week.

18. It is projected that \( t \) years from now the population of a certain country will be changing at the rate \( e^{0.0t} \) million per year. If the current populations is 50 million, what will the population be 10 years from now?
19. An investment will generate income continuously at the constant rate of $1,200$ per year for 5 years. If the prevailing annual interest rate remains fixed at 5\% compounded continuously, what is the present value of the investment?

20. A manufacturer of machinery parts determines that $q$ units of a particular piece will be sold when the price is $p = 124 - 2q$ dollars per unit. The total cost of producing those $q$ units is $C(q)$ dollars, where

$$C(q) = 2q^3 - 59q^2 + 4q + 7,600$$

(a) How much profit is derived from the sale of the $q$ units at $p$ dollars per unit? [Hint: First find the revenue $R = pq$; then find profit = revenue - cost.]

(b) For what value of $q$ is profit maximized?

(c) Find the consumers’ surplus for the level of production $q_0$ that corresponds to maximum profit.