Changes in Migration Patterns of the Capelin as an Indicator of Temperature Changes in the Arctic Ocean

Björn Birnir Sven Þ Sigurðsson Baldvin Einarsson
Alethea Barbaro

Department of Mathematics and Center for Complex and Nonlinear Science
University of California, Santa Barbara
in collaboration with
Marine Research Institute, Reykjavík

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Outline

The capelin

Model

A History of these Models

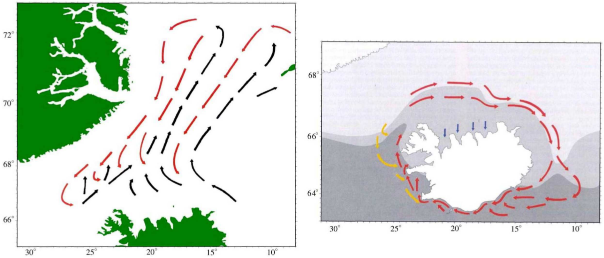
The environment

DEB

The Icelandic capelin (Mallotus villosus) is a species of pelagic fish which is very important to both the ecosystem and the economy.

Icelandic capelin spends the first 2-3 years of its life in the waters to the north of Iceland, along the edge of the continental shelf.

Every year a portion of the stock undertakes a feeding migration to the feeding grounds near Jan Mayen. They then return to the Icelandic waters north of Iceland in October and November and undertake a spawning migration to spawning grounds in the south of Iceland.



The capelin spawn in February-March and then die.

Mathematical Model

We let

$$\mathbf{q}_k(t) = (x_k(t), y_k(t))^\mathsf{T}$$

be the position of particle k and $v_k(t)$ it's speed at time t.

Next, we look at three zones around each particle, determining how neighboring particles affect the particle. (Based on Aoki, 1982, and Huth and Wissel, 1992)

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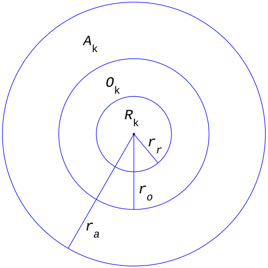
The zones are:

R_k, *Repulsion*,

O_k, *Orientation* and

A_k, *Attraction*.

- Let $|\cdot|$ be the
- number of particles
- in the above zones.



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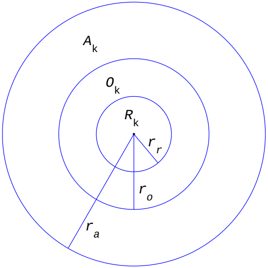


Particle k responds to another particle within...

- the zone of repulsion by heading away from that particle
- the zone of orientation by adjusting it's directional heading to the other particle's directional heading
- the zone of attraction by heading towards the particle

When many particles are withing these zones, these factors have to be weighed together.

Particle *k* updates it's position by

$$\begin{pmatrix} x_k(t+\Delta t) \\ y_k(t+\Delta t) \end{pmatrix} = \begin{pmatrix} x_k(t) \\ y_k(t) \end{pmatrix} + \Delta t \cdot v_k(t+\Delta t) \frac{\mathbf{D}_k(t+\Delta t)}{\| \mathbf{D}_k(t+\Delta t) \|} + \mathbf{C}(\mathbf{q}_k(t))$$

- Here $\mathbf{D}_k$ is the directional heading of particle *k*
- The current **C**, only depends on the position of particle *k* and and is independent of its directional heading (see picture below)
- The speed is the average speed of particles within the zone of orientation. Later we will let it depend on the roe maturity of each fish.
- Time step Δt

The directional heading is updated as

$$\mathbf{D}_k(t+\Delta t) := \left((1-\beta) \underbrace{\begin{pmatrix} \cos(\phi_k(t+\Delta t)) \\ \sin(\phi_k(t+\Delta t)) \end{pmatrix}}_{\text{Reaction to neighbors}} + \beta \underbrace{\frac{\nabla r\left(T\left(\mathbf{q}_k(t)\right)\right)}{\| \nabla r\left(T\left(\mathbf{q}_k(t)\right)\right) \|}}_{\text{Reaction to temperature}} \right).$$

The reaction to the neighbors is as follows:

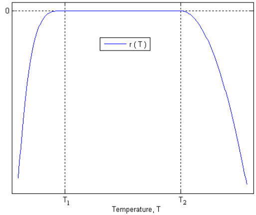
$$\begin{pmatrix} \cos(\phi_k(t+\Delta t)) \\ \sin(\phi_k(t+\Delta t)) \end{pmatrix} = \frac{\mathbf{d}_k(t+\Delta t)}{\| \mathbf{d}_k(t+\Delta t) \|}$$

where

$$\mathbf{d}_k(t+\Delta t) := \frac{1}{|R_k|+|O_k|+|A_k|} \left(\sum_{r \in R_k} \frac{\mathbf{q}_k(t)-\mathbf{q}_r(t)}{\| \mathbf{q}_k(t)-\mathbf{q}_r(t) \|} + \sum_{o \in O_k} \begin{pmatrix} \cos(\phi_o(t)) \\ \sin(\phi_o(t)) \end{pmatrix} + \sum_{a \in A_k} \frac{\mathbf{q}_a(t)-\mathbf{q}_k(t)}{\| \mathbf{q}_a(t)-\mathbf{q}_k(t) \|} \right).$$

The function r describes the reaction to the temperature, T. The interval $[T_1, T_2]$ is the *preferred temperature range* which a particle tends to head into. Later, this range will depend on the roe content of each particle.

$$r(T) := \begin{cases} -(T - T_1)^4 & \text{if} \quad T \leq T_1 \\ 0 & \text{if} \quad T_1 \leq T \leq T_2 \\ -(T - T_2)^2 & \text{if} \quad T_2 \leq T \end{cases}$$



The CV models

Who did what and when

- Called the *CV models*
- Introduced in Vicsek et al. (1995)
- Analyzed and developed in Czirók et al. (1997), Czirók et al. (1999) and Czirók and Vicsek (2000)
- Originated as particle models in statistical mechanics
- Have been applied to herds of mammals, swarms of locusts, and schools of fish (Vicsek et al., 1995; Czirók et al., 1997; Couzin et al., 2002; Buhl et al., 2006)

The CV models

Who did what and when

- Aoki (1982) and Huth and Wissel (1992) employed three sensory zones
- Fish interact differently with each neighbor depending on the distance to the neighbor (Partridge and Pitcher, 1980; Partridge, 1982)
- Fish tend to both aggregate and avoid collisions when traveling (Viscido et al., 2004, 2005; Grünbaum et al., 2005)
- In the zone of orientation the fish adjust both their direction and speed to adjust to other fish around them (Hubbard et al., 2004; Magnússon et al., 2004b)

The CV models

Who did what and when

- No blind region behind the fish, unlike Aoki (1982); Huth and Wissel (1992); Couzin et al. (2002); Kunz and Hemelrijk (2003); Viscido et al. (2004, 2005); Hemelrijk and Kunz (2005); Kunz et al. (2006); Hemelrijk and Hildenbrandt (2008)
- Birnir (2007) analyzed the continuous time limit of this type of model and found several solutions
- In Barbaro et al. (2007), solutions to both models presented in Birnir (2007) were verified numerically
- With sensory zones added, the discrete model exhibits rich behaviour and swarming solutions induced by noise were found

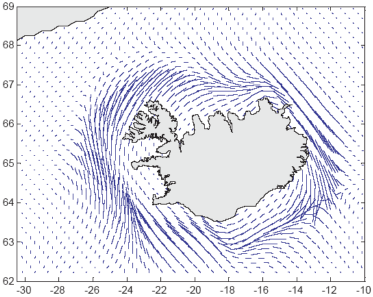
The CV models

Who did what and when

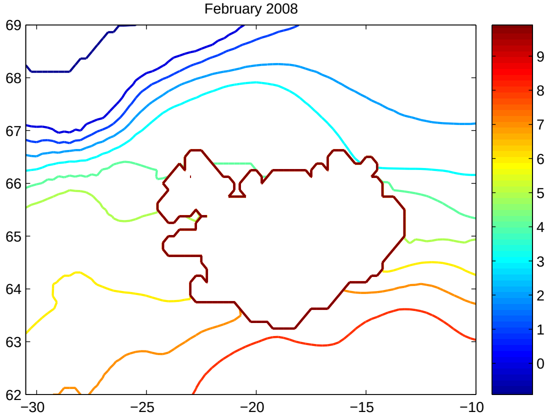
- There was a basic problem with the CV model as modified by Hubbard, Babak, Sigurðsson, Magnússon, and Einarsson (2004), MCV models
- They were computing with relative few particles and had to introduce a homing instinct to reproduce the migrations
- Alethea Barbaro solved this in her PhD (2008) thesis, she computed with thousands of particles, then the schools were able to sense their environment
- A lot of work has gone into modeling a separate capelin stock in the Barent Sea, Fiksen et al. (1995), Huse et al. (1999), Huse (2001), Sigurðsson et al. (2002), Huse et al. (2004) and Magnússon et al. (2004a). Grimm et al. (2006)

The environment

The simulated currents go clockwise around Iceland. The particles do not sense the current, i.e. it only translates them. The maximum speed of the current is 15 km/day, similar to the particles' swimming speed.



The temperature from February 2008:



1984–1985

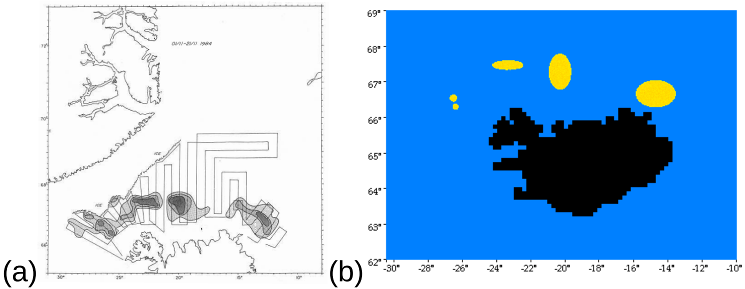


Figure: The distribution of capelin in November of 1984. a) Acoustic data from November 1 to November 21. b) Initial distribution for the simulation.

1984–1985

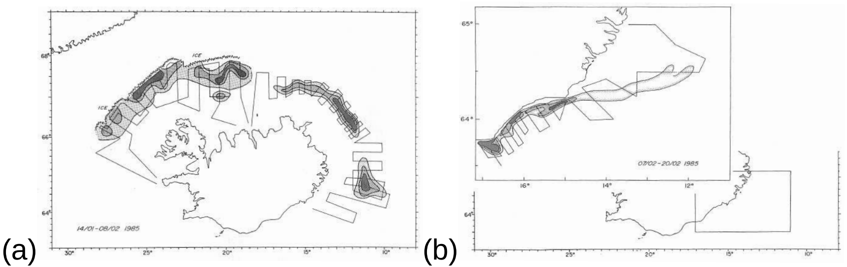


Figure: The distribution of capelin in mid-January to early February of 1985. a) Acoustic data from January 14 to February 8. b) Close up of the distribution of capelin from February 7 to February 20 of 1985.

A sensitivity analysis in our recent paper (Einarsson *et al.*, 2009) showed that two parameters are key to recreating the migration route from a given year: β, the relative weight a particle places on the temperature term to determine its next directional heading, and $[T_1, T_2]$, the particles' preferred temperature range. With thousands of particles, a school effectively produces a global map of the environment.

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Dynamic Energy Budget

We take the model for the weight and internal reserves from Nisbet *et al.*, 2000. Following Hendrata and Birnir (2008), we non-dimensionalize the system and arrive at the following set of ODEs:

$$\dot{W}(t) = \frac{\tilde{\nu}\,[E(t) - W(t)]}{3W_m\,[g + E(t)]} \tag{1}$$

$$\dot{E}(t) = \frac{\tilde{\nu}\,[f(t) - E(t)]}{W_m W(t)} \tag{2}$$

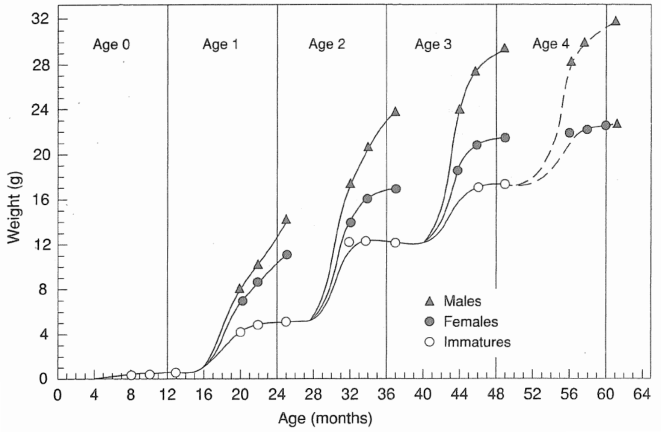
where W is the ratio of the weight of the animal to the maximum weight and E is the ratio of the weight of internal reserves to the maximum weight of internal reserves.

The following ODE for the roe content is derived in Alethea's Ph.D. thesis:

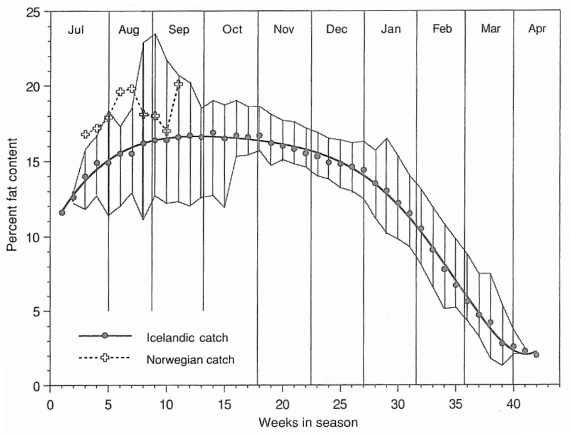
$$\dot{R}(t) = cR(t)^b \left(e^{\left[\frac{T_A}{T} - \frac{T_A}{T_1}\right]} \right) \tag{3}$$

where R is measured in grams, c and b are constants, T_A is the species-specific Arrhenius temperature (K), T is absolute temperature (K), and T_1 is a chosen reference temperature (K), which we in our model take to be the temperature of the surrounding ocean (K).

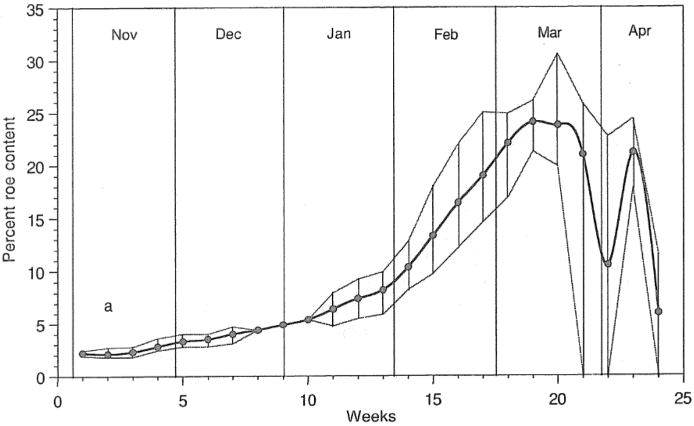
By November, the capelin taking part in the spawning migration have already grown to their full weight. We therefore take the initial weight ratio to be $W(0) = 1$



We also take the internal reserves to begin at the maximum, so $E(0) = 1$.



The initial roe weight is taken to be $R_0 = 0.46$g.



We incorporate the DEB into the model by letting the speed of a particle be determined by the roe maturity. To this end, let

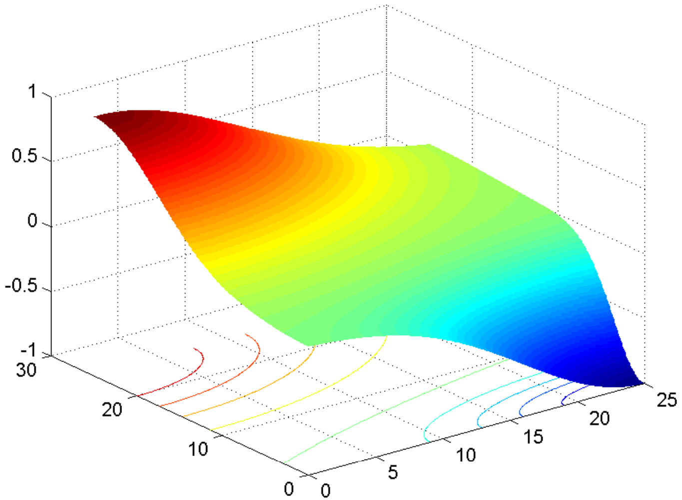
$$\nu_k(t) = \frac{1}{|O_k(t)|} \sum_{j \in O_k(t)} v_j(t) \tag{4}$$

and then

$$v_k(t + \Delta t) = \nu_k(t) + \Gamma\left(\nu_k(t), R_k(t)\right) \tag{5}$$

where Γ determines the acceleration of a particle.

The Γ function:



The capelin are sensitive to oceanic temperature and are known to stay in colder water until their roe content reaches over 10%. We therefore let the maturity level of each fish determine the preferred temperature range of each fish. Once the roe content reaches 10% we trigger the change. The behaviour of a school of particles is influenced greatly by the interactions, not all particles need to have matured so that the school (or part of it) runs into warmer waters.

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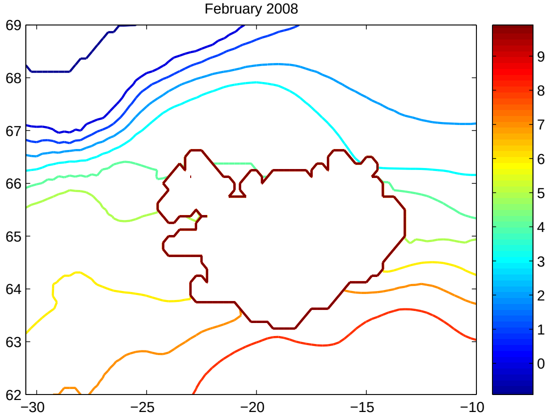

Here is a movie from February of last year (without the DEB). It was used to make a prediction of the spawning migration.

Global Change

With the DEB incorporated we

- Get a more accurate picture of the migration
- Have the timing of the runs

Then we will use the 40 year of acoustic data and current measurements from the Marine Science Institute in Reykjavík to

- Test by simulations temperature changes in the ocean
- Test by improved simulations of the current, changes in the flow of Atlantic waters into the Arctic

Thanks to our collaborators at the Marine Institute of Iceland:

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