Nonlinear Friction Laws in Earthquake Simulations

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Outline

Early Investigation of Empirical Friction Laws

- The late 1970s saw an increased interest in stick-slip instabilities present in laboratory rock experiments as a means of understanding earthquake ruptures.
- Laboratory measurements suggested both the velocity dependence of dynamic friction and time dependence of static friction - resulting in the need for a new framework in which to understand rock friction.
- Dieterich, Ruina, Rice and others used these experiments as a means to formulate constitutive laws capable of describing the frictional stress when rocks were sheared against each other or over a surface [Dieterich (1978), Ruina (1983)].

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Early Investigation of Empirical Friction Laws

Rate and State Friction:

$$\mu = \mu_o + a \ln(\frac{V}{V_o}) + b \ln(\frac{V_o \theta}{D_c}) \tag{1}$$

- μ_0 is appropriate constant for steady-state slip at velocity V_o,
- V is the slip rate
- D_c is the critical slip distance (also denoted by L)
- a and b are associated frictional parameters
- θ is the state variable (average contact lifetime)

Early Investigation of Empirical Friction Laws

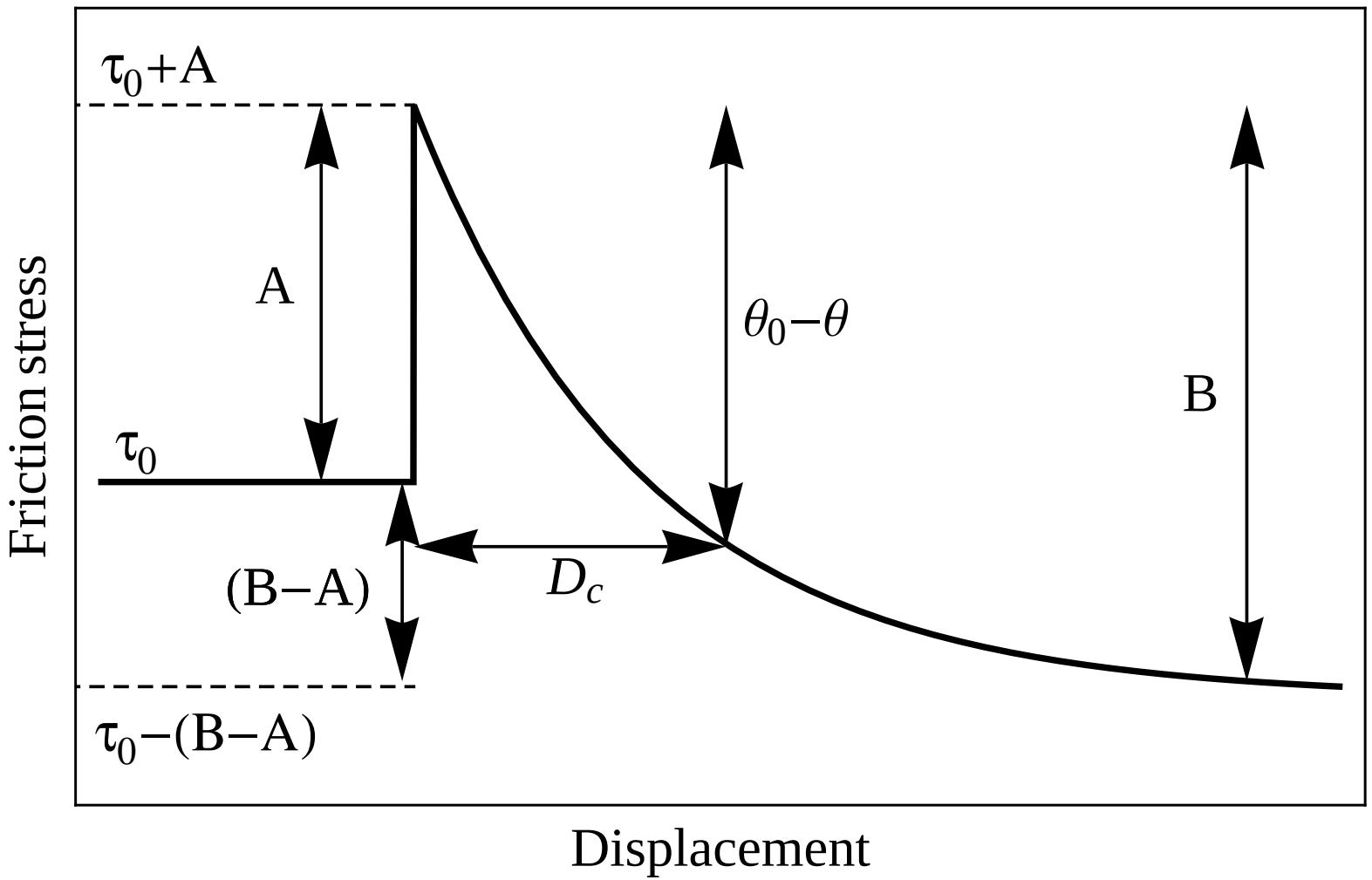
Coupled with:

$$\text{Aging Law:} \quad \frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c} \tag{2}$$

or

$$\text{Slip Law:} \quad \frac{d\theta}{dt} = -\frac{V\theta}{D_c} \ln\left(\frac{V\theta}{D_c}\right) \tag{3}$$

Schematic Diagram

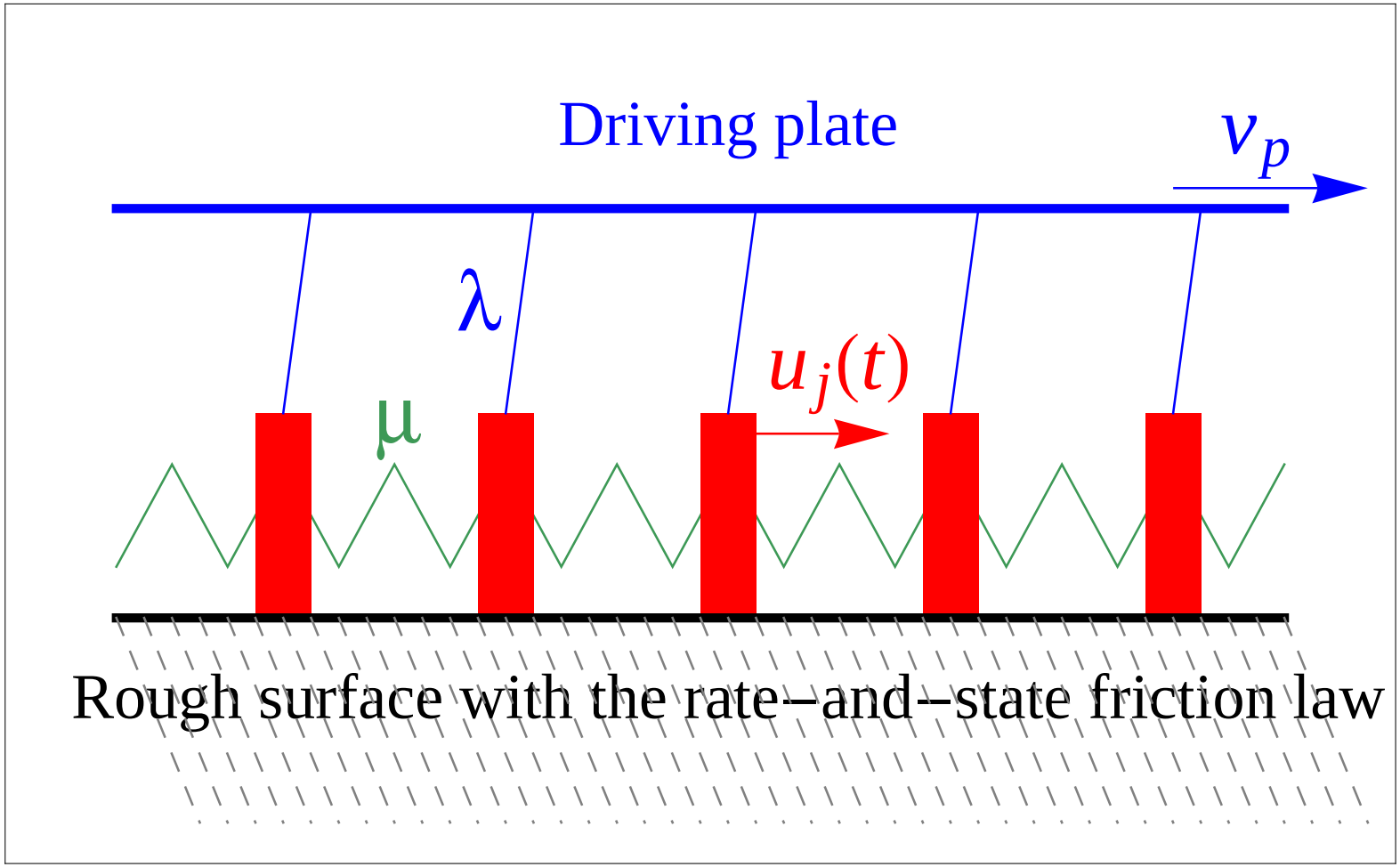


The Burridge-Knopoff Model

- Dynamic Modeling Requirements:
  - Friction law (values for the frictional parameters - somewhat unknown!)
  - Initial spatial distribution of the stress and strength of the material over the fault surfaces.
  - Mathematical description of how these properties evolve during the rupture process.

- One type of dynamic model, studied extensively since its introduction in the 1960s, is the Burridge-Knopoff (1967) model.

The Burridge-Knopoff Model



There are a Number of Modeling Challenges

- The nonlinearity of rate and state friction laws impose challenges in the numerical simulations.
- A correct description of the spatio-temporal variability of parameters involved in the earthquake rupture process is required, but difficult.
- Friction laws like these are derived from small-scale experiments which evokes the question of scaling the parameters to seismic faulting.

The Single Spring-Block Model

The governing equations:

$$\left.\begin{aligned} \dot{u} &= v - v_0 \\ \dot{v} &= (-1/M)[ku + \theta + A\ln(v/v_0))] \\ \dot{\theta} &= -(v/D_c)(\theta + B\ln(v/v_0)) \end{aligned}\right\}, \tag{4}$$

where u the slip relative to the driver plate, θ is the state variable, v the block's velocity, A, B are the stress parameters and D_c is the characteristic slip distance.

The Single Spring-Block Model

The non-dimensional equations of motion are:

$$\left.\begin{aligned} \dot{u} &= v - 1 \\ \dot{v} &= -\gamma^2[u + (1/\xi)(\theta + \ln(v))] \\ \dot{\theta} &= -v(\theta + (1+\epsilon)\ln(v)) \end{aligned}\right\}, \tag{5}$$

where

$$\xi = (kD_c)/A$$

is the nondimensional spring constant,

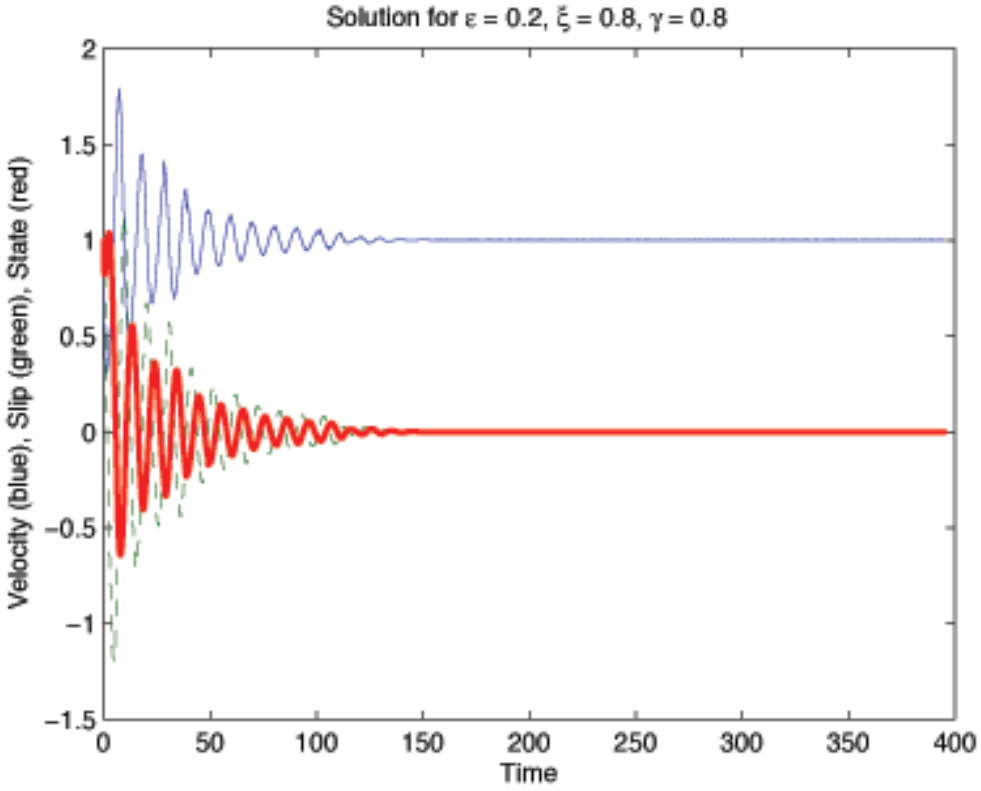
$$\gamma = \sqrt{k/M}(D_c/v_o)$$

is the nondimensional frequency and

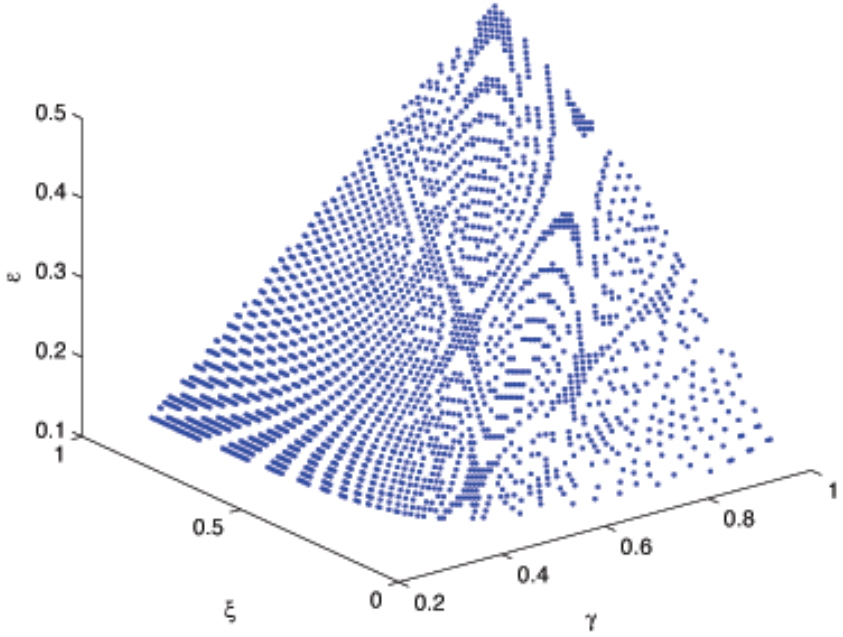
$$\epsilon = (B - A)/A$$

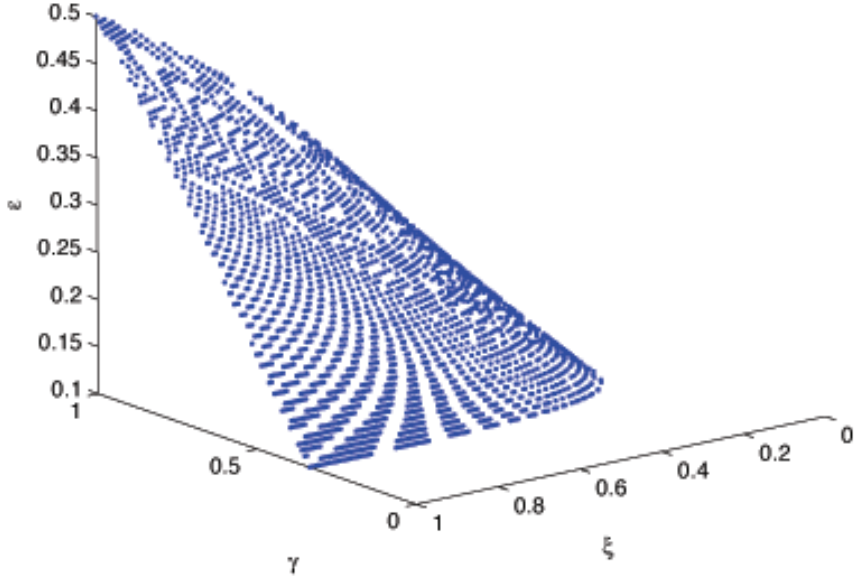
is a ratio of the stress parameters in the RS friction law.

The Stationary Solution

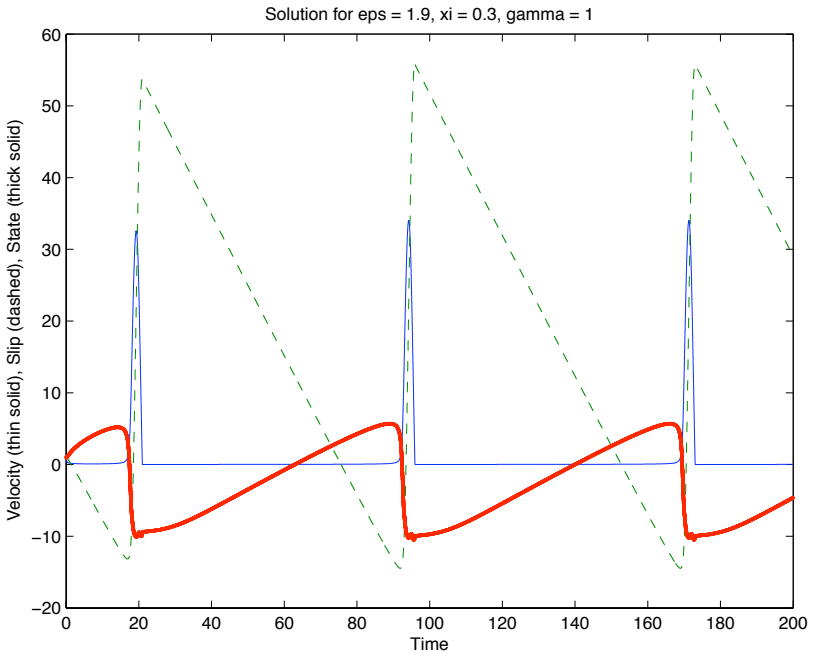


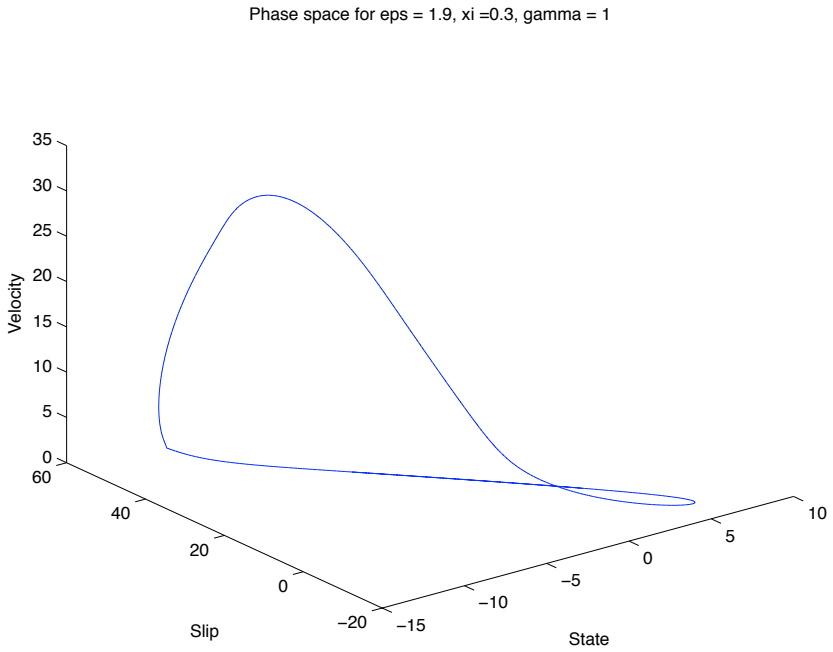
Hopf Bifurcation Parameter Spaces





Periodic Solutions





The Discrete Setting

Governing Equations:

$$\left.\begin{aligned} \ddot{u}_j &= \gamma^2(u_{j-1} - 2u_j + u_{j+1}) - \tilde{\gamma}^2 u_j - (\gamma^2/\xi)(\theta_j + \ln(\dot{u}_j + 1)) \\ \dot{\theta}_j &= -(\dot{u}_j + 1)(\theta_j + (1+\epsilon)\ln(\dot{u}_j + 1)) \end{aligned}\right\} \quad (6)$$

where u_j is the non-dimensional slip of the jth block relative to the driver plate,

$$\gamma = \sqrt{\mu/m}(D_c/Vo) \text{ and}$$

$$\tilde{\gamma} = \sqrt{\lambda/m}(D_c/Vo)$$

are the nondimensional frequencies,

$$\xi = (\mu D_c)/A$$

is the nondimensional spring constant, and

$$\epsilon = (B - A)/A$$

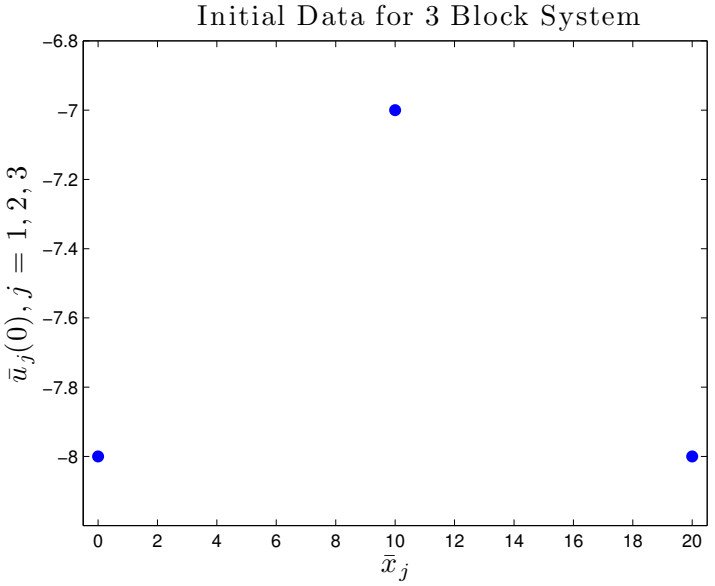
The Discrete Setting

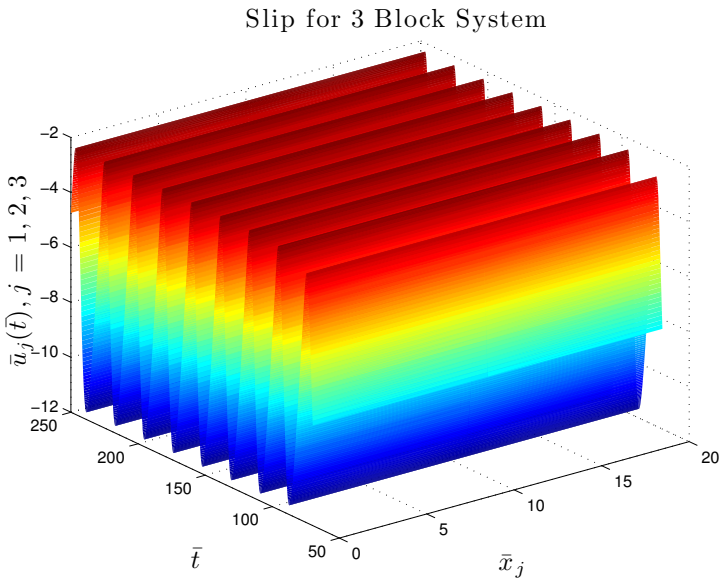
N blocks are evenly spaced on a chain of length 20 units. The initial data is a smooth Gaussian pulse centered at the middle block:

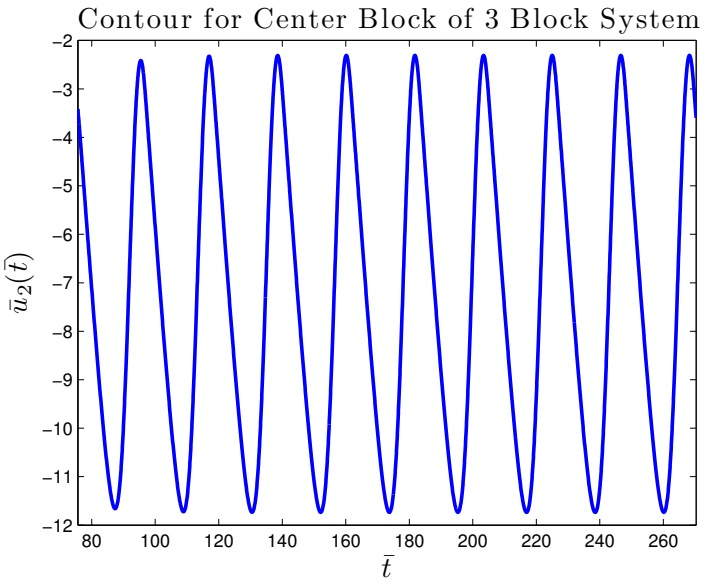
$$u_0(j) = 1.5e^{\frac{-(x_j-10)^2}{\sigma^2}}, \text{ for } j = 1, \ldots N, \text{ where } \sigma = 1,$$
$$v_0(j) = 0, \text{ for } j = 1, \ldots N$$

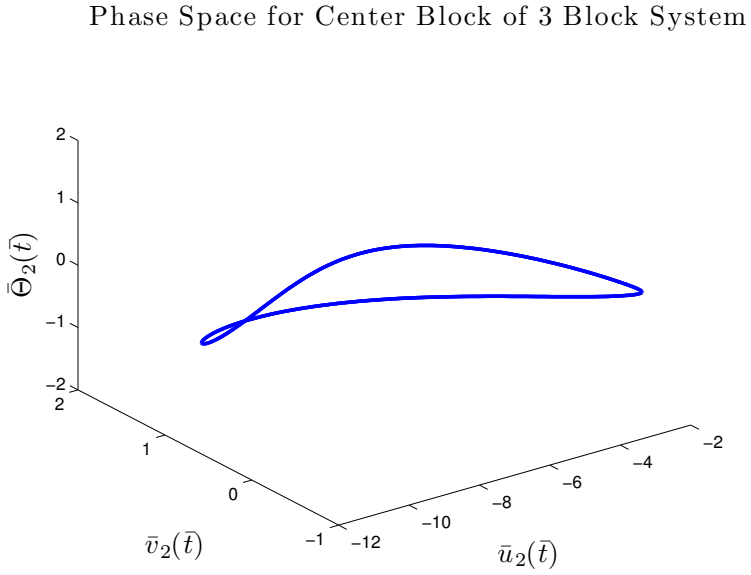
chosen to represent localized departure from the equilibrium regime.

3 Block System

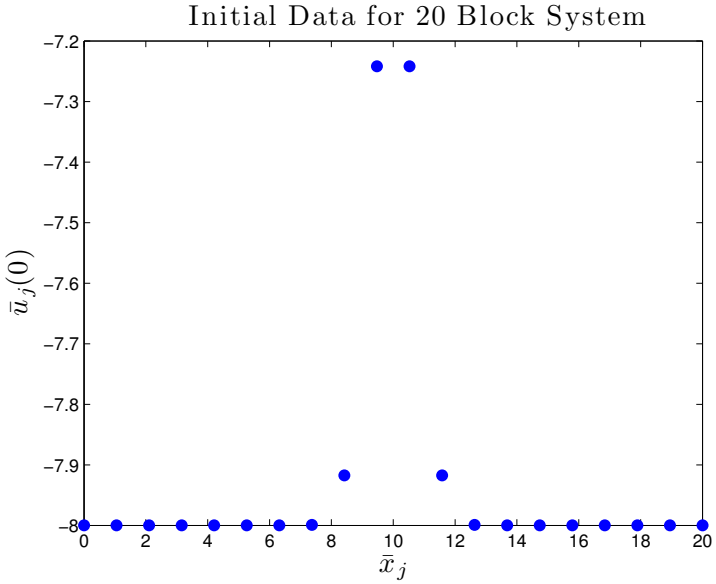


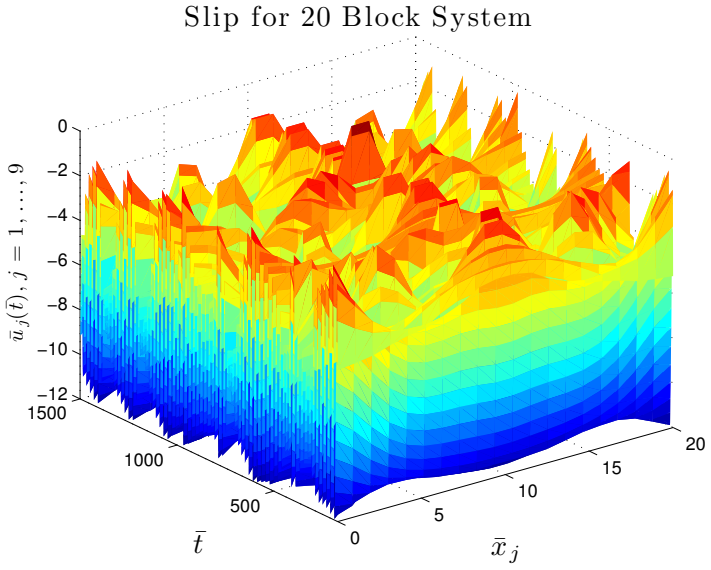


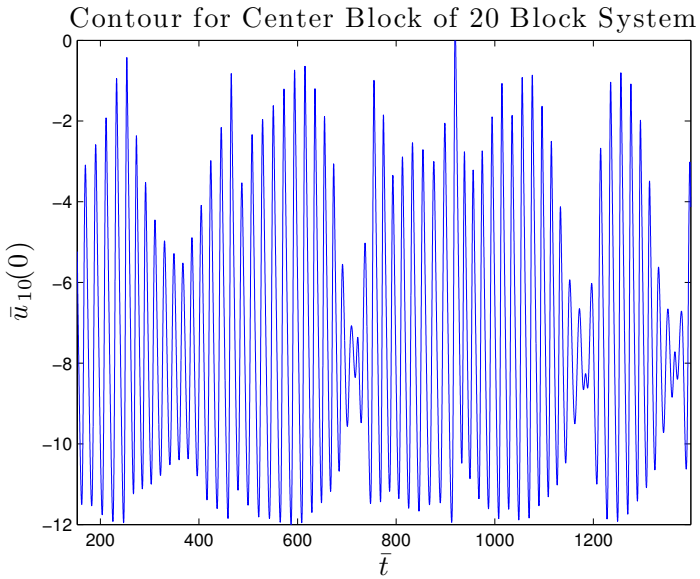


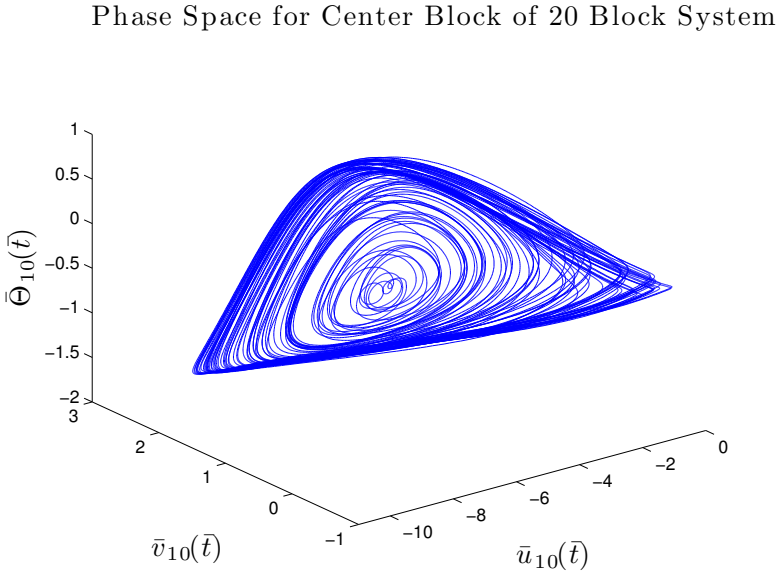


20 Block System









The Continuum Model

The wave equation for $u(x,t)$ driven by rate-and-state friction and its associated state variable evolution equation:

$$\left.\begin{aligned} u_{tt} &= c^2\Delta u - \tilde{\gamma}^2 u - (\gamma^2/\xi)(\theta + \log(u_t+1))) \\ \theta_t &= -(u_t+1)(\theta + (1+\epsilon)\log(u_t+1)) \end{aligned}\right\}, \tag{7}$$

where the final equations now involve a fourth internal parameter:

$$c^2 = \lim_{m,\Delta x\to 0}(\mu D_c^2 \Delta x^2)/(mV_o^2),$$

the square of the wave speed.

Also exhibits chaotic solutions when varying ϵ.

Localized Solutions

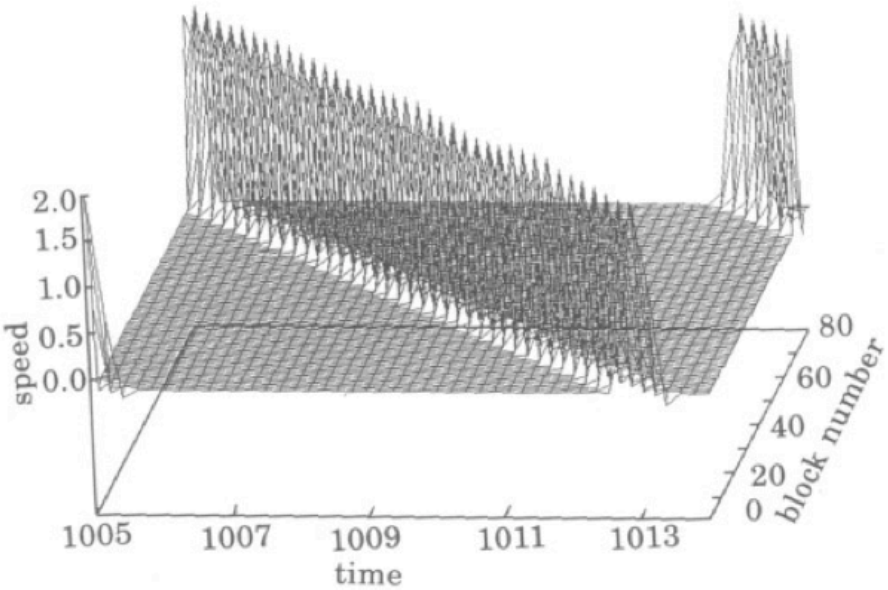
In certain parameter regimes both the discrete and continuous formulations exhibit localized solutions.

These solutions exhibit behavior similar to that of either a *soliton* or a *breather*.

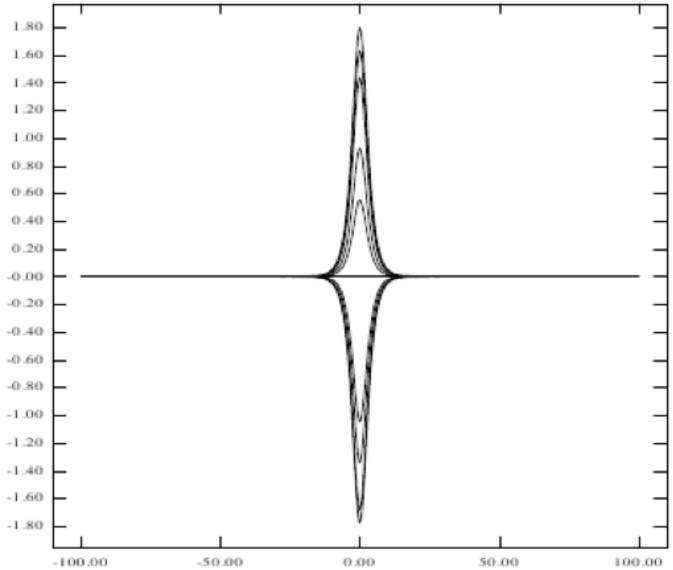
Solitons and Breathers

- The general definition of a *soliton* solution to a nonlinear wave equation is that it has 3 properties:
  - a wave with permanent form
  - localized in space for each fixed point in time
  - if two solitons meet, their forms are preserved after the interaction.
- A *breather*, on the other hand, is a time-periodic, exponentially decaying (in space) solution of a nonlinear wave equation.

Solitons and Breathers



[Schmittbuhl et. al. (1993)]



[Birnir (1994)]

Significance of Solitary Wave Solutions

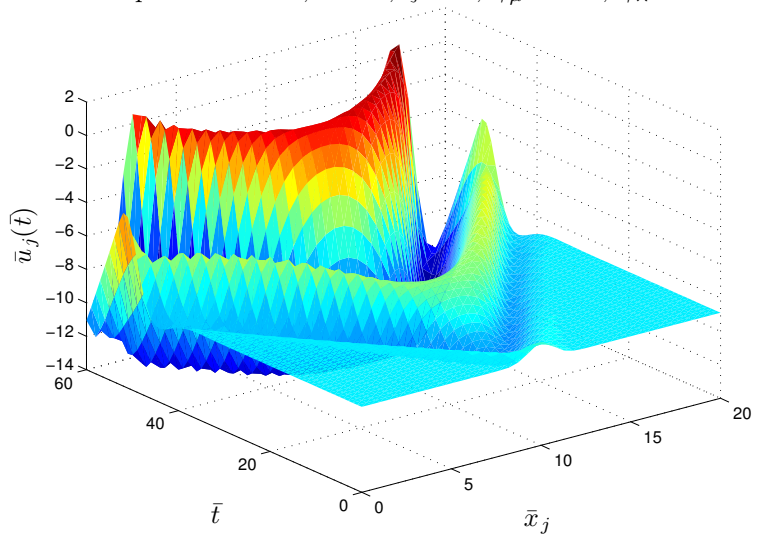
The significance of these types of solitary wave solutions was emphasized by Heaton (1990), who studied dislocation time histories generated from models derived from earthquake waveforms.

Solitons and Breathers

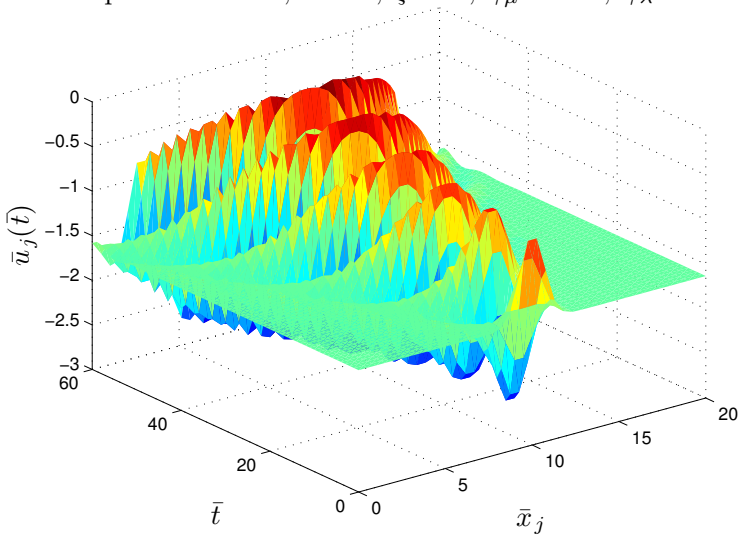
- Similar to the discoveries made by Schmittbuhl et. al. (1993) and Español (1994), we have also seen solitary wave and localized solutions in both the discrete and the continuous models with RS friction.
- These solutions can be understood as a proxy for the propagation of the rupture front across the fault surface during an earthquake and may determine a range for suitable parameter values to be used in dynamic modeling of earthquakes.

Localized Solutions in the Discrete Formulation

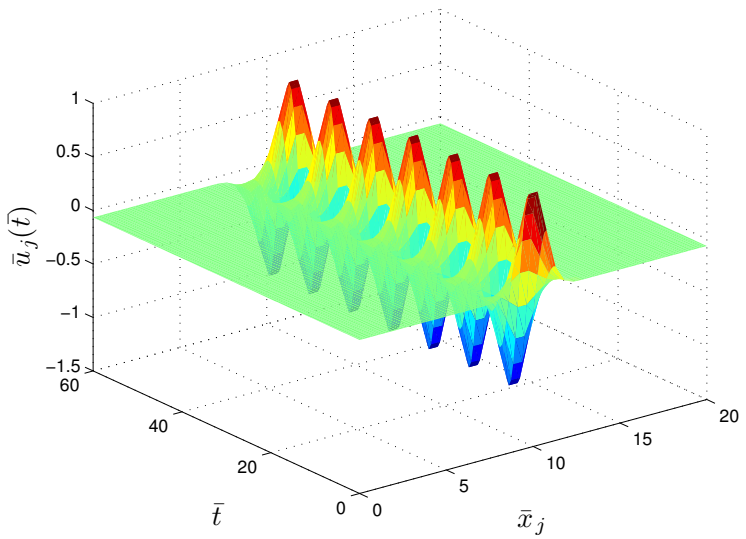
Slip for $N = 40$, $\epsilon = 1$, $\xi = 1$, $\gamma_\mu = 0.5$, $\gamma_\lambda = 0.3$



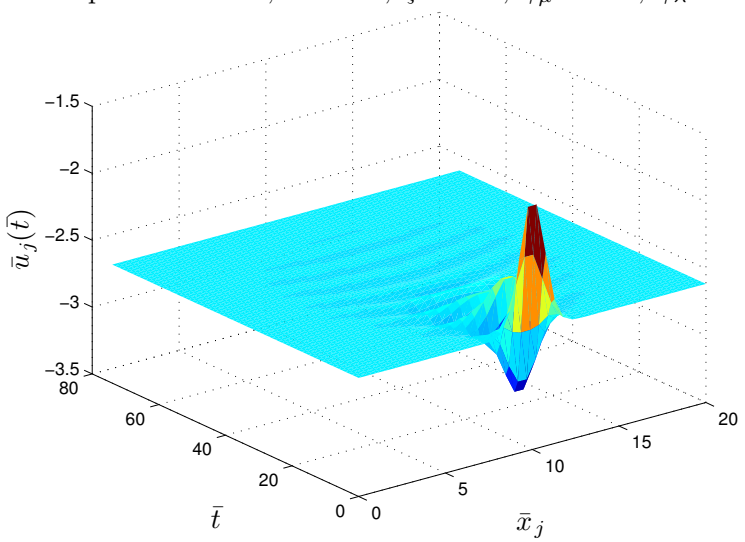
Slip for $N = 40$, $\epsilon = 1$, $\xi = 1$, $\gamma_\mu = 0.5$, $\gamma_\lambda = 0.7$



Slip for $N = 40$, $\epsilon = 1$, $\xi = 1$, $\gamma_\mu = 0.1$, $\gamma_\lambda = 0.7$

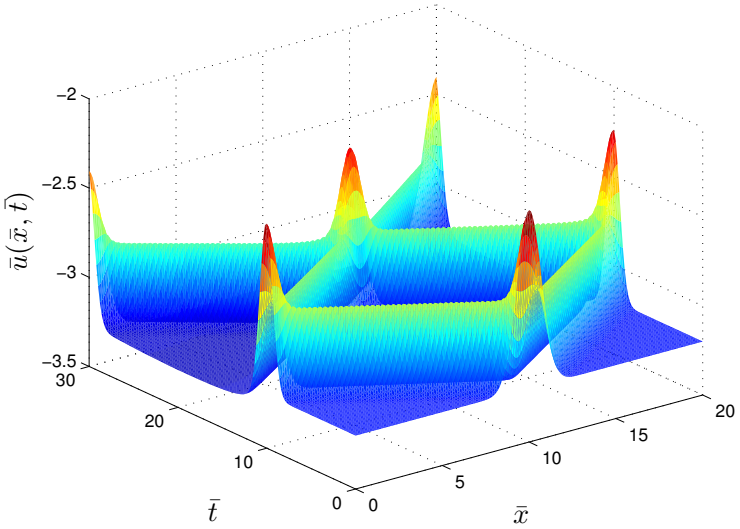


Slip for $N = 40$, $\epsilon = 0.1$, $\xi = 0.6$, $\gamma_\mu = 0.7$, $\gamma_\lambda = 1$

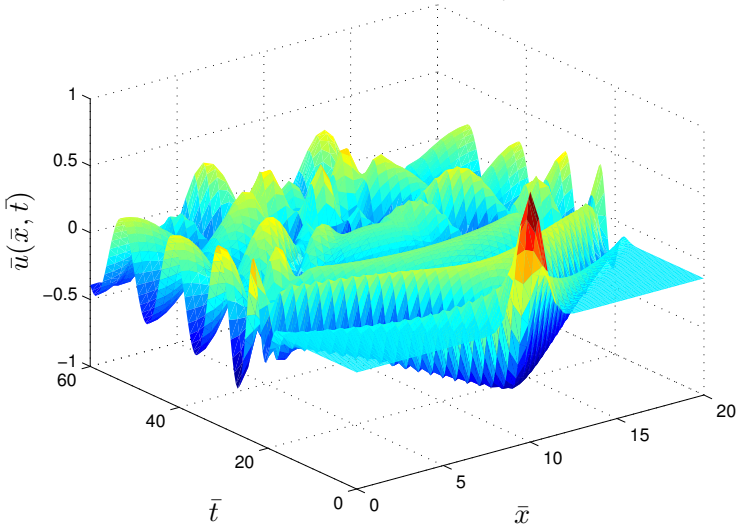


Localized Solutions in the Continuous Formulation

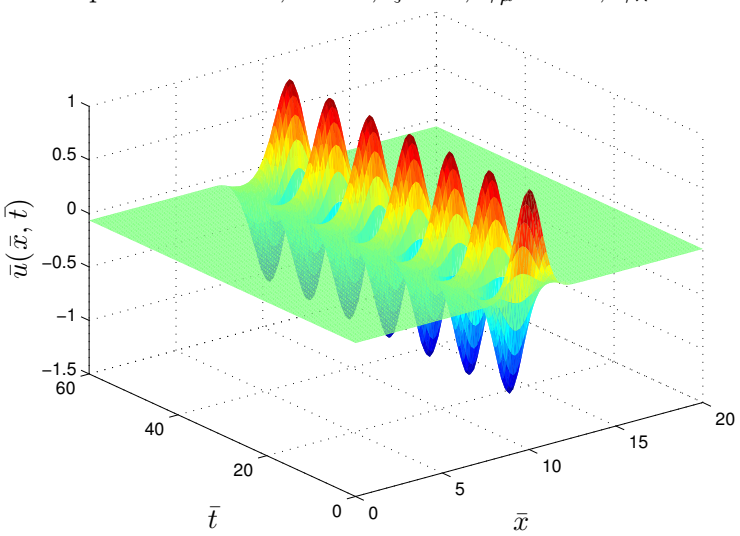
Slip for $c = 1$, $\epsilon = 1$, $\xi = 1$, $\gamma_\mu = 0.1$, $\gamma_\lambda = 0.1$



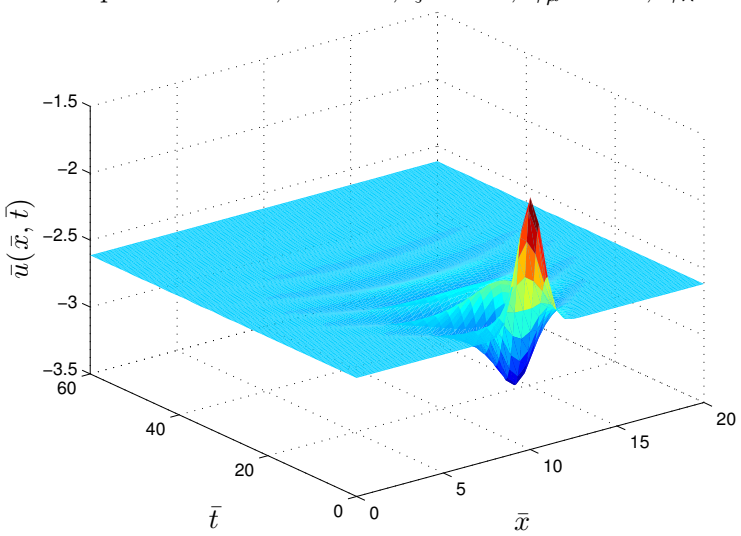
Slip for $c = 0.5$, $\epsilon = 1$, $\xi = 1$, $\gamma_\mu = 0.1$, $\gamma_\lambda = 0.5$



Slip for $c = 0.05$, $\epsilon = 1$, $\xi = 1$, $\gamma_\mu = 0.1$, $\gamma_\lambda = 0.7$



Slip for $c = 0.5$, $\epsilon = 0.1$, $\xi = 0.6$, $\gamma_\mu = 0.7$, $\gamma_\lambda = 1$



Current Status of Dynamic Rupture Modeling

- What are the initial conditions on fault prior to an earthquake????

Elastodynamics

The equations governing anti-plane deformation in 2-d:

$$\rho \frac{\partial^2 u}{\partial t^2} = G(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}),\ \sigma_{xz} = G\frac{\partial u}{\partial x},\ \sigma_{yz} = G\frac{\partial u}{\partial y} \tag{8}$$

solved in the domain $(x, y) \in [0, L] \times [0, H]$, where u is the displacement, σ_{xz} and σ_{yz} are the shear stress components and ρ and G are the density and shear modulus. The boundary conditions are:

$$\sigma_{xz}(0, y, t) = \sigma_n f(V, \theta), \quad y \in [0, H_s] \tag{9a}$$

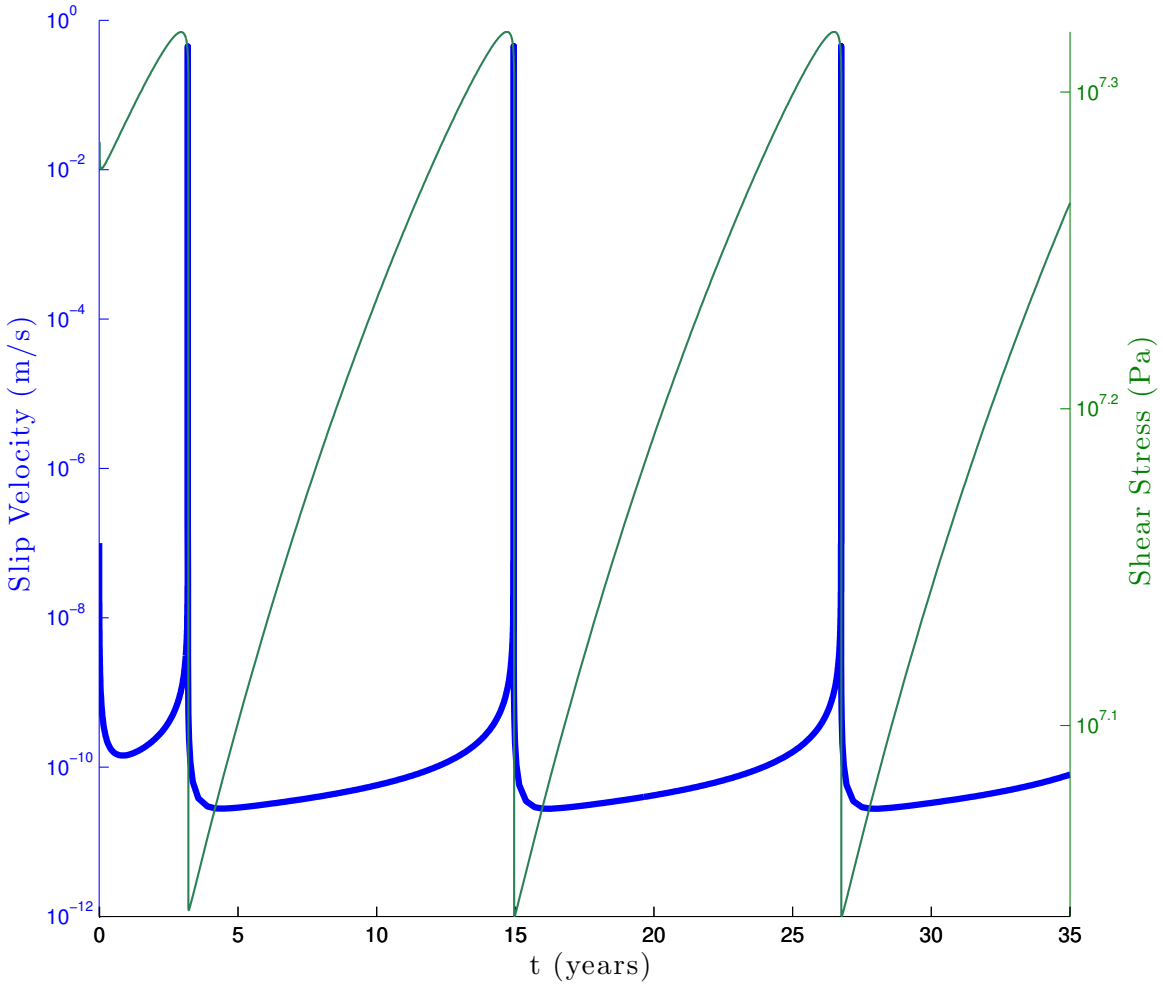
$$u(0, y, t) = V_p t, \quad y \in (H_s, H] \tag{9b}$$

$$\sigma_{yz}(x, 0, t) = 0, \quad x \in [0, L] \tag{9c}$$

$$\sigma_{xz}(L, y, t) = 0, \quad y \in [0, H] \tag{9d}$$

$$\sigma_{yz}(x, H, t) = 0, \quad x \in [0, L] \tag{9e}$$

Switching Method



Conclusions (and some questions!)

- That the transition to chaos for the discrete and continuum model with this friction law is dependent on N and on model parameters (e.g. ϵ) may indicate that when implementing it in dynamic rupture models, qualitative behavior may be lost or altered when considering models at larger scales.
- Localized solutions may suggest a possible range for parameters that could be used in future earthquake modeling.
- Earthquakes are the result of processes in the earth's crust that have evolved over multiple scales in both time and space and we need to account for the evolution of the fault due to tectonic loading in the interseismic period.
- More sources of complexity? Geometry?

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